7.2.1

Monday, October 18, 2021 8:21 AM

IM 2
Classwork...

7.2.1 What does independence tell me?

Conditional Probability and Independence

	Y	N
M	5	10
F	8	13

When two events can occur, either simultaneously or one after the other, how can you calculate the probability of one of them occurring when you already know the other has happened? You will investigate this question today. Then you will use **conditional probabilities** to determine if two events are independent of each other.

7-67. EIGHT THE HARD WAY

Maribelle is playing the board game Eight the Hard Way with her friends. Each player rolls two dice on his or her turn and moves according to the sum on the dice. However, if a player rolls two fours (called "eight the hard way"), that player instantly wins the round and a new round is started.

Delaney steps into the kitchen to get snacks when she hears Maribelle shout "*Woo Hoo! I got an eight!*"

Delaney knows Maribelle rolled a sum of eight. With your team, help Delaney investigate the probability that Maribelle rolled two fours and won the round of play. In other words, calculate the **conditional probability** that Maribelle rolled two fours, given that you know she rolled a sum of eight.

a. Examine the diagram at the right that represents all of the possible sums of numbers when rolling two dice.

First dice

Second dice	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b. Since you know that Maribelle rolled a sum of eight, you do not need to consider all of the outcomes in the sample space. You only need to consider the outcomes with a sum of eight. In the diagram, **highlight all of the ways a sum of eight can be rolled**. How many different ways can a sum of eight be rolled?

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c. You are interested in the event {eight the hard way}. How many different ways can two fours be rolled?

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d. What is the probability of the event {eight the hard way} given that you know that Maribelle rolled a sum of eight?

$\frac{1}{5}$

e. Becca rolls "high" (meaning that she rolls a sum of nine or more). What is the conditional probability that her roll is also an odd number?

$\frac{6}{10} = \frac{3}{5} = .6 = 60\%$

7-68. At Einstein High School (EHS), data on members of the Student Council was collected:

	Eats at Cafeteria	Does not eat at cafeteria	Total
On Student Council	6	34	40
Not on Student Council	30	170	200
Total	36	204	240

a. This type of table is called a **two-way table** and is often used to organize information and calculate probabilities.

What is the probability of a student being on Student Council at EHS?

$$P\left(\text{student council}\right) = \frac{40}{240} = \frac{4}{24} = \frac{1}{6} = .1667 = 16.67\%$$

b. Shade the cells with students who eat at the cafeteria. What is the conditional probability of a student being on Student Council, given that you know the student eats at the cafeteria?

$$P\left(\text{Student Council Given eat at Cafeteria}\right) = \frac{6}{36} = \frac{1}{6} = .1667 = 16.67\%$$

c. Two events, A and B, are **independent** if knowing that event B occurred does not change the probability of event A occurring. That is, two events, A and B, are independent if $P(A) = P(A \text{ given } B)$. Are the events {on Student Council} and {eats at cafeteria} independent? Why or why not?

$$P\left(\text{student council}\right) \stackrel{?}{=} P\left(\text{Student Council Given eat at cafeteria}\right)$$

$$16.67\% = 16.67\% \checkmark$$

$$\boxed{\text{Independent Events}}$$

d. Two events are **mutually exclusive** (or **disjoint**) if they cannot both occur at the same time. That is, two events are mutually exclusive if $P(A \text{ and } B) = 0$. Are the events {on Student Council} and {eats at cafeteria} mutually exclusive? Why or why not?

Not mutually exclusive!
There are 6 students on student council AND eat at cafeteria.

7-69. The following data was collected about students in Mr. Rexinger's high school statistics class.

	Wearing Jeans	Not wearing jeans	Total
Has long hair	7	7	14
Does not have long hair	5	13	18
Total	12	20	32

a. Mr. Rexinger is playing a game with his students. He randomly chooses a mystery student from his class roster. If a player guesses the hair length of the mystery student correctly, the player gets an early-lunch pass. Madeline is the next player. To have the greatest chance of winning an early-lunch pass, should she guess that the student has long hair? Explain.

$P(\text{long hair}) = \frac{14}{32} = .4375 = 43.8\%$

No, less than half have long hair.

b. Mr. Rexinger tells Madeline that the mystery student is wearing jeans. Would you advise Madeline to change her guess? Explain.

$P(\text{long hair Given wearing jeans}) = \frac{7}{12} = .583 = 58.3\%$

c. In a previous course, you may have studied the **association** of two *numerical* variables. Associations between *categorical* events, like having long hair or wearing jeans, are determined by independence—**if two events are independent, then they are *not* associated.**

Are the events {having long hair} and {wearing jeans} associated for the students in Mr. Rexinger's class today? Explain the independence relationship using $P(A) = P(A \text{ given } B)$.

a. $P(\text{long hair}) \stackrel{?}{=} P(\text{long hair Given wearing jeans})$ b.

$43.8\% \neq 58.3\%$

$\boxed{\text{Associated}}$

d. Are the events {having long hair} and {wearing jeans} mutually exclusive? Explain.

not mutually exclusive!

7-70. At Digital Technical Institute, the following data was collected:

	Member of an Honor society	Not a member of an Honor society	Total
On academic probation	0	40	40
Not on academic probation	30	170	200
Total	30	210	240

a. Are the events {on academic probation} and {member honor society} associated at this institute?

P(A) = P(A given B)

P(academic probation) = P(academic probation Given honor society)

$$\frac{40}{240} \neq \frac{0}{30}$$

$\boxed{\text{Associated}}$

b. Are the events {on academic probation} and {member honor society} mutually exclusive at this institute? How many outcomes are in the intersection of the two events?

P(academic Probation AND honor society) = 0