Chapter 3/7 Toolkits

Monday, September 27, 2021 10:13 AM

Chapter 3 and 7 Too...

Chapter 3 and 7 Toolkits

Name ______________________________

Probability Vocabulary and Definitions (2.1.4)

Outcome: A possible result from a probability situation	**Event:** a set of outcomes from a probability situation
Sample Space: A list of all possible outcomes from a situation	**Probability:** The liklihood that an event will occur.

Experimental Probability:

$$\text{Experimental Probability} = \frac{\text{number of successful outcomes in the experiment}}{\text{total number of outcomes in the experiment}}$$

Example: I rolled the die 12 times and 2 came up three times. The experimental probability is P(2) = $\frac{3}{12} = \frac{1}{4} = .25 = 25\%$

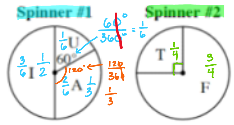
Theoretical Probability:

$$\text{Theoretical Probability} = \frac{\text{number of successful outcomes (events)}}{\text{total number of possible outcomes}}$$

Example: A fair die has six sides. One of the sides has the number 2. The theoretical probability of rolling a 2 on a fair die is P(2) = $\frac{1}{6} = .167 = 16.7\%$

Independent Events (2-118) Two events are Independent if Knowing that one event Occured DOES NOT affect the probability of the other event occuring.	**Dependent Events** Two events are Dependent if Knowing that one event occured DOES affect the probability of the other event occuring.
Example: Flipping five heads in a row, and then flipping another head	Example: Drawing two aces from a deck of cards (without replacing them), and then drawing another ace

Probability Models (3.1.4)



Area Model

Spinner #1

Spinner #2

	$\frac{1}{2}$ I	$\frac{1}{6}$ U	$\frac{1}{3}$ A
$\frac{1}{4}$ T	IT $= \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$	UT $= \frac{1}{6} \cdot \frac{1}{4}$ $\frac{1}{24}$	AT $= \frac{1}{3} \cdot \frac{1}{4}$ $\frac{1}{12}$
$\frac{3}{4}$ F	IF $= \frac{1}{2} \cdot \frac{3}{4}$ $\frac{3}{8}$	UF $\frac{1}{6} \cdot \frac{3}{4}$ $\frac{3}{24} = \frac{1}{8}$	AF $= \frac{1}{3} \cdot \frac{3}{4}$ $\frac{3}{12} = \frac{1}{4}$

Tree Diagram

Spinner #1 Spinner #2

- $\frac{1}{2}$ I
  - $\frac{1}{4}$ T IT $= \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$
  - $\frac{3}{4}$ F IF
- $\frac{1}{6}$ U
  - $\frac{1}{4}$ T UT
  - $\frac{3}{4}$ F UF
- $\frac{1}{3}$ A
  - $\frac{1}{4}$ T AT
  - $\frac{3}{4}$ F AF

Unions, Intersections, and Complements (3.1.5)

If you randomly choose an integer from 1 to 10 the sample space is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} 10
The event "Prime Numbers" would be the set {2, 3, 5, 7}. We will call this Event 1 4
The event "Even Numbers" would be the set {2, 4, 6, 8, 10}. We will call this Event 2 5

Complement	Intersection ×	Union +
The complement is the set of all outcomes in a sample space not in the original event.	The intersection of two events is the event in which both the first event and second event occur.	The union of two events is the event in which the first event OR the second event (or both) occur.
$P(\text{not prime \#}) = 1 - P(\text{Prime \#})$ $= 1 - \frac{4}{10} = \frac{10}{10} - \frac{4}{10} = \frac{6}{10}$ 60%	$P(\text{Prime \# AND Even \#})$ $\frac{4}{10} \cdot \frac{5}{10} = \frac{20}{100}$ 20%	$P(\text{prime \# OR even \#})$ $\frac{8}{10} = 80\%$ {2, 3, 5, 7, ~~2~~, 4, 6, 8, 10} 1 2 3 4 5 6 7 8

Addition Rule for Unions / OR statements

P(A or B) = P(A) + P(B) – P(A and B) OR P(A union B) = P(A) + P(B) – P(A intersection B)

$$P(\text{prime or even}) = P(\text{prime}) + P(\text{even}) - P(\text{prime AND even})$$

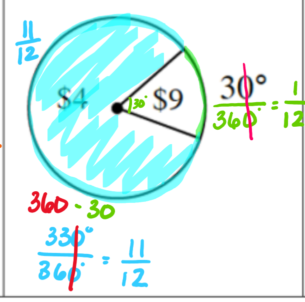
$$= \frac{4}{10} + \frac{5}{10} - \frac{1}{10} = \frac{8}{10} = 80\%$$

Expected Value (3.2.1)

Expected Value

$$\frac{1}{12}(\$9) + \frac{11}{12}(\$4)$$

$$\frac{9}{12} + \frac{44}{12} = \frac{53}{12} \approx \$4.42$$



Fair Game

When the expected value is 0!

Conditional Probability (7.2.3)

Calculating probability when some information about the event is already known.
To calculate **Conditional Probability**

$$P(A \text{ given } B) = \frac{P(A \text{ and } B)}{P(B)}$$

3+3 $\frac{1}{36}$ 1+5 5+1 2+4 4+2 3+3 $\frac{5}{36}$

Example: Calculate the **conditional probability** of rolling a *'hard'* 6 given a 6 was rolled.

A B P(hard 6 | roll 6)

P('hard' 6 given a 6 was rolled) = $\frac{P(\text{hard } 6)}{P(6)} = \frac{1/36}{5/36} = \frac{1}{\cancel{36}} \cdot \frac{\cancel{36}}{5} = \boxed{\frac{1}{5} = 20\%}$

Mutually Exclusive (7.2.2)

P(A and B) = 0 Events that **CANNOT** happen at the same time.

A O O B no overlap between events

Example: What is the probability of having naturally blonde hair and having naturally black hair? Are the events *mutually exclusive*?

0, Yes, mutually exclusive blond black

Independent Events (7.2.3)

P(A) = P(A given B)

Events are independent if knowing one does NOT change the probability of the other event occurring.

Example: Are the events {on Student Council} and {eats in the cafeteria} independent? In other words, does eating in the cafeteria change the probability that a person is in Student Council?

	Eats at cafeteria	Does not eat at cafeteria	Totals
Not on Student Council	30	170	200
On Student Council	6	34	40
Totals	36	204	240

P(A)
P(on Student Council) = $\frac{\cancel{40}}{\cancel{240}} = \frac{1}{6}$

P(A Given B)
P(on Student Council given eats in the cafeteria) = $\frac{6}{36} = \frac{1}{6}$

The probabilities are the SAME so, eating in the cafeteria doesn't change the probability of being in Student Council. The events are **independent**. There is NO association between them.

Association (7-69c)

P(A) ≠ P(A given B)

a relationship between 2 variables or events. When events are NOT independent, we say they are ***associated***.

Example: Are the events {having long hair} and {wearing jeans} associated for this class? In other words, does knowing someone is wearing jeans change the probability that they don't have long hair?

	Wearing jeans	Not wearing jeans	Totals
Has long hair (below chin)	7	7	14
Does not have long hair	5	13	18
Totals	12	20	32

P(A)
P(long hair) = $\frac{14}{32} = \frac{7}{16} = 43.8\%$

P(long hair given wearing jeans) = $\frac{7}{12} = 58.3\%$
P(A Given jeans)

The probabilities are DIFFERENT so, knowing the student is wearing jeans changed the probability that the student didn't have long hair. The events are **associated**!